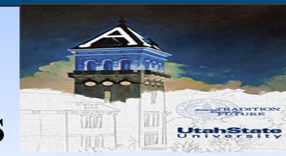
Drought Analysis Based on Copula



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Introduction

Background

Numerical studies shows that global warming is predicted to be the cause of a massive drought that will threaten the lives of millions and take over half the land surface on our planet in the next 100 years. Instead of using traditional univariate analysis for drought assessment, a better approach for describing drought characteristics is to derive the joint distribution of drought variables.

What is Drought ?

Although there is not a universal definition of drought, in the most general sense, drought can be defined with different disciplinary perspectives. In this study we only focus on meteorological drought and defined it using Standardized Precipitation Index (Figure)

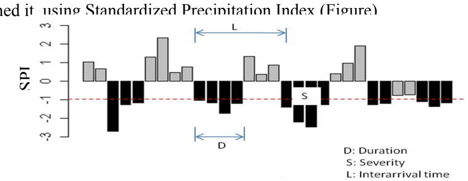


Figure 1. Definition of drought characteristics

Research Objectives

This study aims to model the joint drought duration and severity distribution using two dimensional copula, and calculated the return period of drought based on the derived copula-based joint distribution.

Methods

SPI:

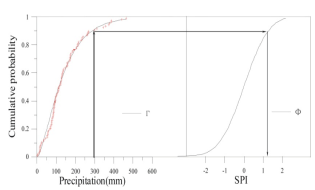
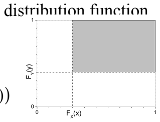


Figure 2 Illustration of the transformation of precipitation (Γdistribution) to SPI (Φ distribution).

Copula: Sklar (1959) showed that for a d-dimensional continuous random variables {x1, …, xd} with joint-CDF H and marginal CDFs uj = Fj(xj), j = 1, …, d, there exists one unique d-copula C such that H(x1, …, xd)= C(u1, …, ud). In this study, Copulas are employed to construct the joint distribution function of drought severity and duration. The reture period is then related to the copula-based distribution function via a conditional distribution function Y

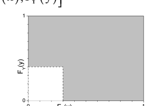
Exceedance probability exceeding x and y :



$$P(X>x \wedge Y>y) = 1 - F_X(x) - F_Y(y) + F_{X,Y}(x,y) = 1 - F_X(x) - F_Y(y) + C(F_X(x), F_Y(y))$$

Return period: $T'_{X,Y} = \frac{1}{P(X \ge x \wedge Y \ge y)} = \frac{1}{1 - F_X(x) - F_Y(y) + C[F_X(x), F_Y(y)]} > \mathrm{Max}[T_X, T_Y]$

Exceedance probability exceeding x or y :



$$P(X>x \vee Y>y) = 1 - F_{X,Y}(x,y) = 1 - C(F_X(x), F_Y(y))$$

Return period: $T'_{X,Y} = \frac{1}{P(X \ge x \vee Y \ge y)} = \frac{1}{1 - C[F_X(x), F_Y(y)]} < \mathrm{Min}[T_X, T_Y]$

Copula Family	*C(u,v)*	Relationship between θ & τ
GH Copula	$C(u,v) = \exp\{-[(-\ln u)^{\theta} + (-\ln v)^{\theta}]^{1/\theta}\}, \theta \in [1,\infty)$	$\tau = 1 - \frac{1}{\theta}, \theta \in [1,\infty)$
Clayton Copula	$C(u,v) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}, \theta \in [1,\infty)$	$\tau = \frac{\theta}{2+\theta}, \theta \in (0,\infty)$
Frank Copula	$C(u,v) = -\frac{1}{\theta}\ln[1 + \frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{(e^{-\theta}-1)}], \theta \in R$	$\tau = 1 + \frac{4}{\theta}[\frac{1}{\theta}\int_0^{\theta}\frac{t}{e^t-1}dt - 1], \theta \in R$

Results

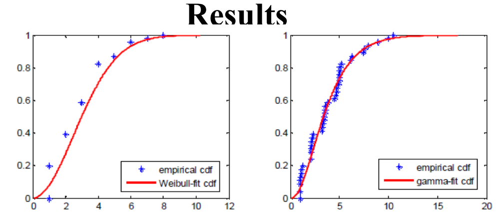


Figure 2 CDFs of drought duration and severity comparing with the empirical distributions

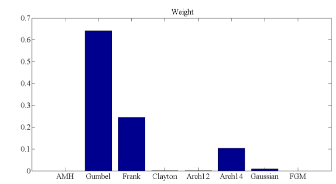


Figure 3 Weight of different Copula family

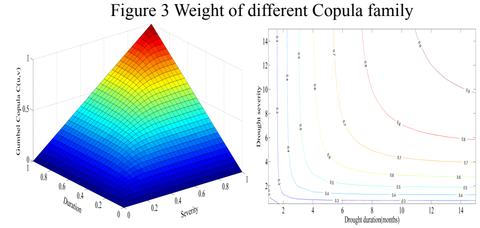


Figure 4 Joint cumulative probability distribution (left figure) and corresponding contour (right figure) of drought duration and severity

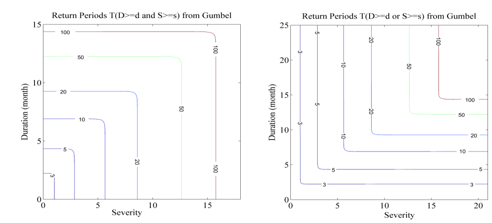


Figure 5 Joint drought duration and severity return period T_{DS} (left figure) and T'_{DS} (right figure) from Gumbel copula

Conclusions

A joint drought duration and severity distribution was constructed in this study. The above results indicate that copulas are a useful tool in exploring the associations of the correlated drought variables.